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VISCOUS FLOW IN THE REGION OF A ROUNDED TRAILING EDGE.(U)
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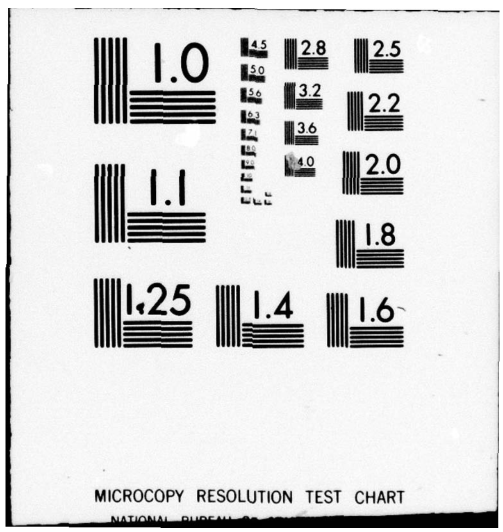
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VISCOUS FLOW IN THE REGION OF A ROUNDED TRAILING EDGE

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31 July 1979

Final Report for Period 1 July 1978 - 30 June 1979

Prepared for

NAVAL AIR SYSTEMS COMMAND
Department of the Navy
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Arlington, VA 22217

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER R79-920003-F	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) 6 Viscous Flow in the Region of a Rounded Trailing Edge		5. TYPE OF REPORT & PERIOD COVERED 9 Final Report 1 Jul[redacted]78 — 30 Jun[redacted]979
		6. PERFORMING ORG. REPORT NUMBER 14 R79-920003-F
7. AUTHOR(s) 10 Ralph/Levy W. Roger/Briley Henry/McDonald		8. CONTRACT OR GRANT NUMBER(s) 15 N00019-78-C-0470
9. PERFORMING ORGANIZATION NAME AND ADDRESS Scientific Research Associates, Inc. P.O. Box 498 Glastonbury, CT 06033		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 12 41
11. CONTROLLING OFFICE NAME AND ADDRESS Naval Air Systems Command Department of the Navy Arlington, VA 22217		12. REPORT DATE 11 31 July 1979
		13. NUMBER OF PAGES 38
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for Public Release; Distribution Unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Rounded trailing edge
Flow separation
Zone embedding
Interaction
Navier-Stokes

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The Interactive Zone Embedding concept is a generalization of concepts which have been used for some time with boundary layer analyses. This concept has been extended to more complex flows using a potential flow for the outer zone and the compressible Navier-Stokes equations for the inner zone. In this interactive manner the flow about a rounded trailing edge was computed for two locations of the outer boundary. The calculations were performed on a 5:1 elliptic cylinder at a Mach number of 0.2 and a Reynolds number based on chord

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of 10^3. A body fitted conformal coordinate system was used. In the first instance the outer boundary was located approximately one chord from the ellipse and in the second the outer boundary was brought to about one-half chord of the ellipse.

The displacement body was chosen as the interactive streamline and the potential flow over this streamline was computed as the sum of the analytic potential flow over the elliptic cylinder plus the flow from a series of sources. The strengths of these sources were determined to make the interaction streamline also be a streamline in the potential flowfield. The resulting potential flow was used to evaluate the static pressure at the outflow boundary of the Navier-Stokes zone.

The calculations were performed with the large Navier-Stokes domain and were repeated with a smaller Navier-Stokes domain. The flow properties in the vicinity of the body for the two calculations were in excellent agreement. Farther from the body the fluid properties were in somewhat poorer agreement. It is anticipated that refinements in the interaction model could improve the flowfield prediction away from the body. In particular, it is felt that locating the inner boundary for the potential flow calculation further from the body and taking care not to bring the downstream boundary of the viscous region too close to the ellipse trailing edge would be very beneficial.

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SUMMARY

The Interactive Zone Embedding concept is a generalization of concepts which have been used for some time with boundary layer analyses. This concept has been extended to more complex flows using a potential flow for the outer zone and the compressible Navier-Stokes equations for the inner zone. In this interactive manner the flow about a rounded trailing edge was computed for two locations of the outer boundary. The calculations were performed on a 5:1 elliptic cylinder at a Mach number of 0.2 and a Reynolds number based on chord of 10^3. A body fitted conformal coordinate system was used. In the first instance the outer boundary was located approximately one chord from the ellipse and in the second the outer boundary was brought to about one-half chord of the ellipse.

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INTRODUCTION

The cornerstone of modern aerodynamics is the ability to predict the lift and drag on lifting surfaces such as airfoils, or cascades of airfoils when concerned with turbomachinery applications. Early aerodynamicists recognized that at high Reynolds numbers, typical of aircraft flight, viscous effects would be small in the main except, as Prandtl pointed out, very close to the airfoil surface within the boundary layer. With this concept it was possible to utilize the highly developed mathematical theory of potential flows to predict the inviscid irrotational pressure field acting on the airfoil or cascade. One flaw remained, however, and that was that an infinity of solutions could be obtained from potential flow theory and that some physical condition had to be invoked to determine which solution was obtained in reality. A highly satisfactory resolution to this problem for incompressible steady flow around airfoils with sharp trailing edges was developed and termed the Kutta-Joukowski condition. This technique amounted to adjusting the circulation about the airfoil or cascade until the rear stagnation point occurred precisely at the sharp trailing edge. Clearly such a technique does not extend to rounded trailing edges and could reasonably be questioned even for sharp trailing edges in unsteady flow. Given the necessity of treating potential flows through cascades of airfoils with rounded trailing edges, the turbomachinery aerodynamicists developed alternative strategies, which will subsequently be discussed. A turbine airfoil typical of modern design is shown in Fig. 1.

At this point it is appropriate to digress momentarily and discuss the origin of rounded trailing edges. In external or wing aerodynamics, it has usually been found structurally feasible and aerodynamically desirable (from a drag point of view) to maintain a sharp trailing edge on the airfoil wherever possible. Some advantage may be gained from blunt trailing edges at transonic or hypersonic speeds but this point will not be dwelt on here. In turbomachines blunt trailing edges arise frequently simply because manufacturing tolerances do not permit aerodynamically sharp trailing edges to be obtained. Chord lengths of one inch or less are frequently encountered in turbomachines, for instance. Secondly, high temperature and rotational environments such as are encountered in turbines do not permit the use of

sharp trailing edges due to resulting stress concentrations which would arise at these sharp points. Trailing edges could be squared off; however, it is often claimed that squared off trailing edges result in a drag penalty relative to rounded trailing edges, and consequently rounded trailing edges have retained their popularity over the years. Typically trailing edge radii can be about one quarter of the leading edge radius and in some applications, particularly small gas turbines, may even approach the leading edge radius in size.

Returning now to the problem of determining a suitable alternative to the Kutta-Joukowski condition for rounded trailing edges, the procedure presently in widespread use for steady flow is based upon cascade data correlations. The overall approach and one data correlation is given in detail by Lieblein (Ref. 1). The essence of the procedure is that the Kutta-Joukowski condition is replaced by a specified turning angle of the flow through the cascade obtained from a data correlation. Camber, solidity and air inlet angle being the essential parameters of the correlation. The correlation is usually presented in terms of a deviation angle which is defined as the difference between the average direction of the exiting flow and the direction of the blade mean line at the trailing edge. Lieblein (Ref. 1) cites the work of Carter and Hughes (Ref. 2) as providing a theoretical basis for such a correlation, and much subsequent work by industry has concentrated on expanding the correlation data base, at considerable cost and with the obvious problems of extrapolation to new designs and different flow regimes. The steady and unsteady transonic cases are, to say the least, somewhat clouded at the present time. Recently, Gostelow (Ref. 3) has reviewed the question of the location of the trailing edge stagnation point for rounded trailing edges and suggested a rather intuitive extra-

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1. Lieblein, S.: Experimental Flow in Two-Dimensional Cascades. Chapter VI, NASA SP 36, Ed., I. A. Johnsen and R. O. Bullock, 1965.
2. Carter, A. D. S. and Hughes, H. P.: A Theoretical Investigation Into the Effect of Profile Shape on the Performance of Aerofoils in Cascade. Aero. Research Council R&M 2384, 1946.
3. Gostelow, J. P.: Trailing Edge Flows Over Turbomachine Blades and the Kutta-Joukowski Condition. University of Cambridge, Dept. of Engineering Report CUED/ATurbo/TR 55, 1974.

polation procedure to replace the deviation angle correlation procedure currently in widespread use. Gostelow's suggestion, typical of the present level of understanding of this problem, is simply to perform a number of potential flow calculations with assumed deviation angles and select the "true" deviation angle as being that angle which allowed the upper and lower surface trailing edge pressures to meet when the pressure is linearly extrapolated from the 85% chord point. Subsequently, Miller and Serovy (Ref. 4) reviewed four of the more recent suggested methods of estimating deviation angles and observed that the suggestion of Gostelow (Ref. 3) and the suggestion of Miller and Servoy (Ref. 4) both gave comparable and fairly good results for a very limited sample of data near the design incidence. Off design, the predictions were not in particularly good agreement with the measured deviation angles. In all the techniques examined, the proposed methods of obtaining the deviation angle were rather intuitive and not at all rigorously based. Miller and Serovy (Ref. 4) concluded that "The general lack of agreement between estimated and measured deviation angles using inviscid calculations suggests that viscous effects must be included in any general deviation angle estimation procedure." Gostelow (Ref. 3) concluded that for the three suggested deviation angle estimating procedures he examined, including his own and the Miller-Serovy (Ref. 4) suggestion, that "All three procedures fail under conditions of high incidence or loading." The published discussion following the Miller-Serovy paper (Ref. 4) clearly indicates a consensus of opinion favoring the development of a more rigorous deviation angle prediction scheme which would incorporate viscous and eventually transonic effects, thus providing much of the impetus behind the present proposal. Earlier attempts to include viscous effects such as that due to Gostelow et al., (Ref. 5) were restricted to unseparated trailing

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4. Miller, M. J. and Serovy, G. K.: Deviation Angle Estimation for Axial Flow Compressors Using Inviscid Flow Solutions. Trans. of ASME J. of Engineering for Power, April 1975.
5. Gostelow, J. P., Lewkowicz, A. K. and Shaalan, M. R.: Viscosity Effects on the Two-Dimensional Flow in Cascades. Aeronautical Research Council of Great Britain Current Paper CP 872, 1967.

edges or used highly simplified treatments of the separated flow region (Geller, Ref. 6).

In order to demonstrate the importance of the stagnation point location (or equivalently, the deviation angle) Gostelow (Ref. 3) presents three calculated pressure distributions around a cascade of airfoils with rounded trailing edges for minor changes in the stagnation point location. The resulting changes in the predicted pressure distribution are unfortunately very significant. Gostelow's results are reproduced in Fig. 2. Also taken from Gostelow's report are the typical predicted deviation angles which follow from the various closure hypotheses as compared with measured values for a particular cascade and these results are given here in Fig. 3. Not mentioned in these more recent studies on deviation angle prediction is the concurrent very poor situation which must exist with regard to loss predictions. One bright aspect is, as Lieblein (Ref. 1) points out, that the effect of compressibility on the deviation angle correlations is not large at least to incidences below the first appearance of sonic velocity. Clearly, however, transonic flow must have a large and poorly understood effect on the flow in the region of the rounded trailing edge.

Before leaving this section and with the foregoing background it is probably appropriate now to define the terminology "rounded trailing edge" a little more precisely since on a microscopic level aerodynamically "sharp" trailing edges could be regarded as blunt. Two scales enter the problem, the first an inviscid one and following the examples shown by Gostelow (Ref. 3) for instance, uncertainties in the stagnation point location of order 0.5% chord create significant changes in the predicted pressure field. Clearly the trailing edge radii would have to be much less than 0.25% chord to tie the stagnation point down sufficiently well that the airfoil trailing edge could be termed "sharp". As a working hypothesis in the subsequent work a trailing edge radius of less than 0.025% chord will be considered

6. Geller, W.: Incompressible Flow Through Cascades With Separation. AGARDograph No. 164, Boundary Layer Effects in Turbomachines, Ed., J. Surugue, 1972.

"sharp". Such a value could be expected to vary somewhat with airfoil shape but serves the present purposes adequately. Experimentally Nash et al., (Ref. 7) and Inoue and Kaneko (Ref. 8) for instance, found that reducing the trailing edge thickness below about 2% chord had little effect on the observed airfoil or cascade performance, respectively. This experimental observation unfortunately does not translate into the fact that the precise location of the stagnation streamline, or equivalently the deviation angle, is unimportant in calculation schemes when the trailing edges are thinner than 2% chord. Gostelow (Ref. 3) has clearly demonstrated the unfortunate sensitivity of the flow predictions for much thinner trailing edges than 2% chord.

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7. Nash, J. F., Quincey, V. C., and Callinan, J.: Experiments on Two-Dimensional Base Flow and Subsonic and Transonic Speeds. Aero. Research Council of Great Britain R&M 3427, 1966.
8. Inoue, M. and Kaneko, K.: Effect of Trailing Edge Thickness on the Cascade Performance of Circular-Arc Blade. 2nd Int. JSME Symposium Proceedings, Tokyo, September 1972.

ANALYSIS

Overview

This study includes the calculation of the flow in the vicinity of the rounded trailing edge of an airfoil. In performing flow field calculations there is a constant desire to improve efficiency by limiting the computational domain such that the available grid points are used only where they are absolutely necessary. Far field boundary conditions for the computational domain are rigorously known only at infinity. Although transformations are available to map grid points to infinity, greater efficiency can be achieved by dividing the flow field into inner and outer zones and by using computational analyses specially suited to each zone. Prandtl's boundary layer theory is a remarkably successful example of this philosophy. The outer layer or zone is presumed to be inviscid and is usually treated as a potential flow which is readily predicted for the given body. The boundary layer or inner zone is then computed with boundary conditions representing the imposed inviscid pressure gradients. Although in some cases interaction between zones is negligible, there are many instances, especially in the cases of flows containing separation, where the interaction must be modeled to maintain accuracy. The general technique of Interactive Zone Embedding is directly extendable to multi-zone, three-dimensional, and time dependent problems.

Related Work

Three decades ago a great deal of progress was made in understanding the interaction of co-flowing streams. Tsien and Finston, Ref. 9, and Lighthill, Ref 10, studied the interaction of an inviscid subsonic stream

9. Tsien, H. S. and Finston, M.: Interaction Between Parallel Streams of Subsonic and Supersonic Velocities. Journal of the Aeronautical Sciences, Vol. 16, No. 9, Sept., 1949.
10. Lighthill, M. G.: Reflection at a Laminar Boundary Layer of a Weak, Steady Distrubance to a Supersonic Stream, Neglecting Viscosity and Heat Conduction. Quarterly Journal of Mechanics and Applied Mathematics, Vol. III, 1950.

along a surface and an outer supersonic stream. Crocco and Lees (Ref. 11) extended this work by studying the interaction of a boundary layer and an outer supersonic stream. The boundary layer was treated as quasi-one-dimensional using integral boundary layer theory and the supersonic flow was modeled by the Prandtl-Meyer relation. Using these simple analyses for the two streams, the rate of growth of the displacement thickness was used to determine the outer flow angle. This angle in turn was used to set the boundary layer static pressure. This technique was used to compute flows in adverse pressure gradients up to the point of separation.

The zone interaction concept has received considerable impetus as efforts have been made to extend boundary layer theory into the separated flow region. Studies by Stewartson (Refs. 12, 13) and others developed triple deck, where the boundary layer is divided into an upper layer which is rotational but inviscid and a lower layer which can be described by the conventional incompressible boundary layer equations. The outer third layer or "free stream" is perturbed by the boundary layer presence according to Lighthill's concept (Ref. 14). The three layers are linked by the pressure field and an iteration devised to develop the mutually compatible interaction. Much of interest about separation and interaction can be learned from triple deck, but as a predictive tool it is restricted to asymtotically high Reynolds numbers and an inner layer where the boundary layer approximations are valid. The relationship between triple deck and numerical studies of interacting boundary layers has been examined by Davis and Werle (Ref. 15). In the

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11. Crocco, L. and Lees, L.: A Mixing Theory for the Interaction Between Dissipative Flows and Nearly Isentropic Streams. Journal of the Aeronautical Sciences, Vol. 19, No. 10, Oct., 1952.
12. Stewartson, K.: Multistructural Boundary Layers. Advances in App. Mech., Vol. 14, 1974, pp. 145-239.
13. Stewartson, K., Williams, P. G.: Self Induced Separation. Proc. Roy. Soc. Ser. A, Vol. 312, 1969, pp. 181-206.
14. Lighthill, M. J.: On Boundary Layers and Upstream Influence. Proc. Roy. Soc. Ser. A, Vol. 217, 1953, pp. 478-507.
15. Davis, R. T., Werle, M. J.: Numerical Methods for Interacting Boundary Layers. Proceedings of the 1976 Heat Transfer and Fluid Mechanics Institute, Stanford University Press, 1976, pp. 317-339.

conventional interacting boundary layer approach, essentially the lower and middle decks of triple deck are merged and solved numerically, together with an upper deck which represents the interaction, and the result has been termed "composite equations". A similar approach was taken by Briley and McDonald (Ref. 16), using both the Navier-Stokes and boundary layer equations for the inner region and a potential flow interaction model to provide outer boundary conditions. By comparison of interacted Navier-Stokes and interacted boundary layer solutions, it was shown in Ref. 16 that for small thin separation bubbles the conventional boundary layer approximations remain valid when streamwise "ellipticity" arising both from reverse flow velocities and subsonic displacement interaction effects are accounted for. Two general approaches for computing zone embedding flow models with interaction have been suggested and both are reviewed from the viewpoint of boundary layer separation by Williams (Ref. 17). In the direct approach (Ref. 16), a time-dependent iteration path is followed to obtain interacted velocity (or pressure gradient) boundary conditions for the inner region which match the outer inviscid flow corrected for displacement interaction with the inner viscous layer. The inner layer equations are solved as a time-dependent boundary layer or Navier-Stokes problem. A pressure correction to some unperturbed inviscid body pressure distribution is computed by a (transient) displacement surface interaction and is imposed as a direct boundary condition for the viscous layer. The computational mesh is adjusted at each time step to encompass the separation region both upstream and downstream and includes the transiently changing boundary layer thickness. In Ref. 16, using the boundary layer approximations, downstream boundary conditions or vorticity are allowed for by taking implicit sweeps normal to the wall and parallel to the wall at each time step. The composite system is then integrated out to steady state.

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16. Briley, W. R., McDonald, H.: Numerical Predictions of Incompressible Separation Bubbles. J. Fluid Mech., Vol. 69, Part 4, 1975, pp. 631-656.
17. Williams, J. C., III: Incompressible Boundary-Layer Separation. Ann. Rev. Fluid Mechanics, 1977.

In the interacted inverse approach Carter (Ref. 18) assumed a distribution of some quantity, in this instance displacement thickness, and solved the (steady) boundary layer equations for the imposed pressure gradient. The difference between the computed pressure gradients obtained from inviscid flow concepts using the assumed or computed boundary layer displacement distribution and the pressure gradients determined from the boundary layer calculations is used to correct the assumed skin friction or displacement thickness distribution. Since in concept the direct (Ref. 16) and indirect (Ref. 18) approaches are merely different strategies using the same general displacement surface pressure interaction approach to obtain solutions for essentially the same steady equations, there should in principle be little difference in the solutions generated. The observation can be made however, that this indirect approach may involve spatial marching of the boundary layer into a region of separated flow and hence either an (expensive) additional iteration must be undertaken to allow for the reverse flow (Refs. 18, 19), or an additional approximation invoked such as "FLARE", neither of which are required in the direct approach. FLARE, an acronym for Flugge-Lotz and Reyner (Ref. 20), stabilizes the computation by taking a small fraction of the absolute value of the axial convective term in the reversed flow in order to forward march into the separated region. As might be expected, Carter (Ref. 18) found that as the recirculation velocities decreased the differences between reverse flow iteration and FLARE decreased for thin separation bubbles using the interacted inverse boundary layer approach. As a corollary to the similarity of the steady state equations solved by the direct and indirect approaches (FLARE excluded), it follows that results obtained with the interacted inverse approach (Ref. 18) which indicate that it deals adequately with the separation singularity problem should be equally valid for the interacted direct approach (Ref. 16).

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18. Carter, J. E.: Inverse Solutions for Laminar Boundary-Layer Flows with Separation and Reattachment. NASA Technical Report, NASA TR R-447, November, 1975.
19. Klineberg, J. M., Steger, J. L.: On Laminar Boundary-Layer Separation. AIAA Paper 74-94, 1974.
20. Reyhner, T. A., Flugge-Lotz, I.: The Interaction of a Shock Wave With a Laminar Boundary Layer. Int. J. Non-Linear Mech., Vol. 3, No. 2, June 1968, pp. 173-199.

In the case of supersonic external flow Levy, Shamroth, Gibeling and McDonald (Ref. 21) solved a shock wave turbulent boundary layer interaction problem by an interaction technique. The inner zone was computed by the steady finite differenced boundary layer equations. The small magnitude of the reversed flow velocities permitted the FLARE stabilization to be used without excessive error. The outer zone was modeled by the Prandtl-Meyer relation applied at the displacement body as the boundary layer was marched downstream. Since neither the forward marching boundary layer solution nor the Prandtl-Meyer relation is elliptic, a shooting technique was used to provide the upstream influence of the shock-boundary layer interaction. The calculations of Ref. 21 were in excellent agreement with the experimental data of Spaid and Frishett (Ref. 22).

Interactive Zone Embedding

Zone Embedding is a technique for increasing computational efficiency by dividing the flow field into two or more regions and using appropriate numerical analyses for each region. An example of zone embedding is the computation of flow about an airfoil by solving the Navier-Stokes equations in the vicinity of the wing with the farfield boundary conditions determined by potential flow about the airfoil.

Interactive Zone Embedding improves the accuracy of zone embedding by accounting for the effect the fluid dynamic processes in each zone have on the fluid in adjacent zones. Three steps are required to implement interactive zone embedding. The first step is to choose the analyses for the different parts of the flow. In regions where the stresses are important they of course must be included in the analysis. However, in regions where the stress is negligible, a simplified analysis is often justified. Knowing the analyses to be used, the second step is the division of the flow field

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21. Levy, R., Shamroth, S. J., Gibeling, H. J., and McDonald, H.: A Study of the Turbulent Shock Wave Boundary Layer Interaction, AFFDL-TR-76-163, Feb. 1977.
22. Spaid, F. W., and Frishett, J. C.: Incipient Separation of a Supersonic Turbulent Boundary Layer, Including Effects of Heat Transfer. AIAA Journal, Vol. 10, No. 7, July 1972.

into zones. The flow in each of these zones is analyzable by a different method. The determination of the zones is subjective and care must be exercised in maintaining consistency with the fluid dynamic analyses to be used. The third and equally crucial step is the determination of coupling relations to carry information between adjacent zones. Each pair of adjacent zones requires a pair of relations to communicate the effect of each zone on its neighbor. The flow of information provided by the coupling relations is shown schematically in Fig. 4. Since the adjacent analyses do not all contain the same amount of information about the fluid, the coupling relations can transmit only the "least common denominator" of information contained in adjacent analyses. For example, modeling the displacement body influence on external flow does not model the heat transfer or the shear at the interface. If these additional properties are negligible at this part of the flow field and if they are not contained in the external flow analysis, the displacement modeling may be well chosen. If these properties are modeled in the external flow, their values at the interface should be included in the interaction model. These coupling relations contain the exchange of information - momentum, mass, energy, vorticity, etc., across an interface.

Interactive Zone Embedding is directly applicable to three-dimensional and time-dependent problems. Navier-Stokes calculations about finite span wings could use this approach to limit the computational domain to the vicinity of the surface. Oscillating airfoil and cascade calculations could limit the Navier-Stokes zone to the vicinity of the airfoils by using a time-dependent potential flow analysis to model the far field wave effects. Clearly the payoff for Interactive Zone Embedding comes from using a very inexpensive outer analysis to restrict the relatively expensive analysis to a confined region.

APPLICATION TO FLOW AT A ROUNDED TRAILING EDGE

Interactive Zone Embedding, as presented in the previous section, was used to analyze the flow at a rounded trailing edge. The Navier-Stokes equations were used in the zone near the airfoil since the boundary layer equations were judged to be inadequate for the relatively large recirculation regions expected. In the outer zone a potential flow was used. The coupling relation from the inner zone to the potential flow was the specification of a streamline, and the coupling relation from the outer zone to the Navier-Stokes was the specification of static pressure. The actualization of this analysis will now be discussed.

To simplify the problem during the initial phase of the study, the flow over the rear of a 5:1 elliptic cylinder was considered. An orthogonal conformal body fitted coordinate system was used consisting of ellipses in the azimuth and hyperbolas in the radial direction as shown in Fig. 5. Since these initial calculations were run at zero angle of attack, symmetric flow was assumed and only the rear upper quadrant of the flow field was computed. The equations relating the computational coordinates y_1, y_2 to the physical plane x_1, x_2 are

$$x_1 = \cosh(y_1)\cos(y_2)$$
$$x_2 = \sinh(y_1)\sin(y_2) \tag{1}$$

Since these coordinates are conformal there is only one metric, h, at each grid point. Sine h must satisfy

$$\int h\,dy_1 = S_1 \, , \int h\,dy_2 = S_2 \tag{2}$$

where S_1, S_2 are the physical distances along the coordinate lines, h can be determined from either

$$h = \left[\left(\frac{\partial x_1}{\partial y_1}\right)^2 + \left(\frac{\partial x_2}{\partial y_1}\right)^2\right]^{1/2} = \left[\left(\frac{\partial x_1}{\partial y_2}\right)^2 + \left(\frac{\partial x_2}{\partial y_2}\right)^2\right]^{1/2}$$

as (3)

$$h = \cosh^2(y_1) - \cos^2(y_2)$$

The Navier-Stokes equations are used in the form:

$$\frac{\partial \rho}{\partial t} = -\frac{1}{h^2}\frac{\partial}{\partial y_1}(h\rho v_1) - \frac{1}{h^2}\frac{\partial}{\partial y_2}(h\rho v_2)$$

$$\frac{\partial \rho v_1}{\partial t} = -\frac{1}{h^2}\frac{\partial}{\partial y_1}(h\rho v_1^2) - \frac{1}{h^2}\frac{\partial}{\partial y_2}(h\rho v_1 v_2)$$

$$+ \rho v_1 v_2 \frac{\partial \frac{1}{h}}{\partial y_1} - \rho v_2^2 \frac{\partial \frac{1}{h}}{\partial y_2} - \frac{\bar{p}}{\bar{\rho}\bar{U}^2}\frac{1}{h}\frac{\partial p}{\partial y_1} + \frac{\bar{F}_1}{Re} \tag{4}$$

$$\frac{\partial \rho v_2}{\partial t} = -\frac{1}{h^2}\frac{\partial}{\partial y_1}(h\rho v_1 v_2) - \frac{1}{h^2}\frac{\partial}{\partial y_2}(h\rho v_2^2)$$

$$+ \rho v_1 v_2 \frac{\partial \frac{1}{h}}{\partial y_2} - \rho v_1^2 \frac{\partial \frac{1}{h}}{\partial y_1} - \frac{\bar{p}}{\bar{\rho}\bar{U}^2}\frac{1}{h}\frac{\partial p}{\partial y_2} + \frac{\bar{F}_2}{Re}$$

where v_1 and v_2 are the velocity components along the coordinate lines, ρ is the density, p is the static pressure, $\bar{p}$, $\bar{\rho}$, and $\bar{u}$ are reference pressure density and velocity, Re is the reference Reynolds number, and F_1 and F_2 are the viscous force components of:

$$F = \nabla \cdot \left[\bar{\bar{\pi}} + \bar{\bar{\pi}}^t\right]$$

where $\bar{\pi}$ and $\bar{\pi}^t$ are the viscous stress tensor and the turbulent average stress tensor respectively.

These equations are solved by the linearized Bock Implicit technique of Briley and McDonald (Ref. 23). The governing equations are replaced by an

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23. Briley, W. R., and McDonald, H.: Solution of the Multidimensional Compressible Navier-Stokes Equations by a Generalized Implicit Method. Journal of Computational Physics, Vol. 24, Aug. 1977, p. 372.

implicit time difference approximation, optionally a backward difference or Crank-Nicolson scheme. Terms involving nonlinearities at the implicit time level are linearized by Taylor expansion about the solution at the known time level, and spatial difference approximations are introduced. The result is a system of multidimensional coupled (but linear) difference equations for the dependent variables at the unknown or implicit time level. To solve these difference equations, the Douglas-Gunn (Ref. 24) procedure for generating alternating-direction implicit (ADI) schemes as perturbations of fundamental implicit difference schemes is introduced in its natural extension to systems of partial differential equations. This technique leads to systems of coupled linear difference equations having narrow block-banded matrix structures which are solved efficiently by standard block-elimination methods.

Symmetry boundary conditions were imposed on the wake centerline. On the solid surface the no-slip condition was imposed along with the second normal derivative of pressure set to zero. At the outer boundary ellipse the normal second derivatives of total pressure and cartesian transverse velocity were set to zero. The static pressure on this boundary was either set to the value determined by potential flow about the 5:1 ellipse (Zone Embedding without interaction) or to the pressure determined by the interactive coupling relation to be discussed below.

Specification of inflow boundary conditions is slightly more complex since the inflow properties are specified through the boundary layer to the wall. The boundary conditions are the specification of total pressure, transverse velocity and the zero first streamwise derivative of static pressure. These conditions in cooperation with the specified static pressure on the outflow boundary set the angle of attack, the mass flux through the computational zone and allow the static pressure at the inflow plane to be determined by the rest of the flow field. Allowing the inflow static pressure to adjust to downstream flow conditions is necessary to allow transient

24. Douglas, J., and Gunn, J. E.: A General Formulation of Alternating Direction Methods. Numerische Math., Vol. 6, 1964, pp. 428-453.

pressure waves to propagate upstream and out of the computational domain. The specification of total pressure rather than streamwise velocity maintains physically reasonable inflow properties as a pressure transient passes through the inflow boundary.

The difference between the known total pressure and the computed static pressure is the dynamic head. However near the wall this quantity approaches zero. If no further modeling is done, an inconsistency develops near the wall as the static and total pressures approach the same values. In fact, even though the wall point on the inflow boundary need not be included in the calculations since it is a corner point, the static and total pressures are equal in the corner. The static pressure boundary condition is used to set a total pressure on the wall which is consistent with the interior of the computational domain. The inflow total pressure in the boundary layer is recomputed each time step to be consistent with the interior of the computational domain. The inflow total pressure in the boundary layer is recomputed each time step to be consistent with the free stream total pressure, the wall pressure and the input dynamic head distribution in the boundary layer as:

$$p_T = (p_{T_\infty} - p_s)f(y/\delta) + p_s$$

where $f(y/\delta) = (\frac{u}{u_e})^2$ is specified on input.

The interaction between the Navier-Stokes zone and the Potential Flow zone starts with the specification of a streamline near the body. In the results presented below the displacement streamline was used. This streamline is found by integrating vertically from the wall at a series of streamwise locations until a preset mass flux has been captured. In order to make this Navier-Stokes determined streamline a streamline of the potential flow, the superposition principle for incompressible potential flow is used. A series of mass sources is placed on the streamline whose strengths are determined to keep the potential flow about the body plus the flow from the sources from crossing the streamline. An additional source was placed on the forebody to account for the flow at the mid-chord between the interaction streamline and the ellipse. The net effect of this source distribution plus the potential flow about the body results in the potential flow about the

body results in the potential flow about the streamline. This new potential flow solution determines the static pressure boundary condition used on the outflow boundary of the Navier-Stokes zone. The interaction procedure was found to require negligible computer time and was repeated after each time step in the Navier-Stokes solution procedure.

RESULTS

Interactive Zone Embedding was used to analyze the flow about a rounded trailing edge. Three test cases were run for laminar flow over the rear of a 5:1 elliptic cylinder at a freestream Mach number of 0.2 and a Reynolds number based on chord of 10^3. Two computational domains were used for the embedded Nqvier-Stokes calculations. The larger domain extended approximately one chord from the body, see Fig. 5, and the smaller domain extended only about one half chord. With the larger domain, calculations were run both with and without interaction to verify that since the interaction pressure correction falls off as $1/r^2$, there was only a very small influence of the boundary layer and wake on the static pressure at the outer computational boundary.

The interacted solutions in the larger and smaller domains were then compared. An overview of these two solutions is first presented by comparing the solution in the smaller domain with the solution in the corresponding portion of the larger domain shown in Fig. 6. Contours of constant horizontal and vertical components of velocity, vorticity, and total and static pressure are compared in Figs. 7-11. Figure 12 shows the velocity vector comparison. Particularly insofar as the recirculation region is concerned the agreement between the larger and smaller domain calculations is very good.

As expected, the static pressure is the most sensitive parameter and, as shown in Fig. 11, it exhibited the most variation. The most important part of the static pressure field is the pressure distribution at the airfoil surface, since this can be integrated to calculate force and moment coefficients. The wall static pressure coefficients for the larger domain, smaller domain, and for the reference potential flow are presented in Fig. 13. The results of the larger and smaller domain are seen to be virtually identical.

The interaction streamlines for the two domains are presented in Fig. 14. These are nearly identical except at the downstream end of the smaller domain where the streamline shape was extrapolated. Outside the computational domain the interaction streamline was assumed to follow a potential flow streamline. The current treatment of the displacement streamline at and beyond the computational domain is suspected to be one source of the discrepancy between the larger and smaller domain results away from the body. Figure 15 shows that

the velocity and static pressure on the wake centerline for the two calculations are in good agreement near the body but that this agreement deteriorates downstream near the outflow boundary.

Traditionally Interactive Zone Embedding had been run with a boundary layer analysis for the inner zone. The displacement body was used for the interaction since it is both a streamline of the flow and a readily identifiable boundary layer parameter. However, in the boundary layer application the static pressure at the displacement body is the same as at the boundary layer edge, (normal pressure gradients negligible in the boundary layer) but this is not necessarily an accurate approximation in the present case. Since the shear layer is not thin there and the geometry nontrivial there are significant static pressure variations beyond the displacement body, and the integration of the normal pressure from the displacement body to the outer computational boundary might not be the same for the (assumed) potential flow as it is for the Navier-Stokes equations. This could contribute to the static pressure at the outflow boundary of the smaller computational domain falling between the values computed by the potential flow and the larger domain Navier-Stokes at the same locations as shown in Fig. 16. Note that Fig. 16 also shows a kink in the smaller domain static pressure in the region of the extrapolated streamline as discussed in the previous paragraph. The effects of choosing a streamline much closer to the outer boundary of the viscous region as the inner boundary for the potential flow requires further investigation. The treatment of the outflow boundary including the extrapolation of the interaction streamline could also be refined.

SUMMARY AND CONCLUSIONS

The Interactive Zone Embedding concept is a generalization of concepts which have been used for some time with boundary layer analyses. This concept has been extended to more complex flows using a potential flow for the outer zone and the compressible Navier-Stokes equations for the inner zone. In this interactive manner the flow about a rounded trailing edge was computed for two locations of the outer boundary. The calculations were performed on a 5:1 elliptic cylinder at a Mach number of 0.2 and a Reynolds number based on chord of 10^3. A body fitted conformal coordinate system was used. In the first instance the outer boundary was located approximately one chord from the ellipse and in the second the outer boundary was brought to about one-half chord of the ellipse.

The displacement body was chosen as the interactive streamline and the potential flow over this streamline was computed as the sum of the analytic potential flow over the elliptic cylinder plus the flow from a series of sources. The strengths of these sources were determined to make the interaction streamline also be a streamline in the potential flowfield. The resulting potential flow was used to evaluate the static pressure at the outflow boundary of the Navier-Stokes zone.

The calculations were performed with the large Navier-Stokes domain and were repeated with a smaller Navier-Stokes domain. The flow properties in the vicinity of the body for the two calculations were in excellent agreement. Farther from the body the fluid properties were in somewhat poorer agreement. It is anticipated that refinements in the interaction model could improve the flowfield prediction away from the body. In particular, it is felt that locating the inner boundary for the potential flow calculation further from the body and taking care not to bring the downstream boundary of the viscous region too close to the ellipse trailing edge would be very beneficial.

REFERENCES

1. Lieblein, S.: Experimental Flow in Two-Dimensional Cascades. Chapter VI, NASA SP 36, Ed., I. A. Johnsen and R. O. Bullock, 1965.
2. Carter, A. D. S. and Hughes, H. P.: A Theoretical Investigation Into the Effect of Profile Shape on the Performance of Aerofoils in Cascade. Aero. Research Council R&M 2384, 1946.
3. Gostelow, J. P.: Trailing Edge Flows Over Turbomachine Blades and the Kutta-Joukowski Condition. University of Cambridge, Dept. of Engineering Report CUED/ATurbo/TR 55, 1974.
4. Miller, M. J. and Serovy, G. K.: Deviation Angle Estimation for Axial Flow Compressors Using Inviscid Flow Solutions. Trans. of ASME J. of Engineering for Power, April 1975.
5. Gostelow, J. P., Lewkowicz, A. K. and Shaalan, M. R.: Viscosity Effects on the Two-Dimensional Flow in Cascades. Aeronautical Research Council of Great Britain Current Paper CP 872, 1967.
6. Geller, W.: Incompressible Flow Through Cascades With Separation. AGARDograph No. 164, Boundary Layer Effects in Turbomachines, Ed., J. Surugue, 1972.
7. Nash, J. F., Quincey, V. C., and Callinan, J.: Experiments on Two-Dimensional Base Flow and Subsonic and Transonic Speeds. Aero. Research Council of Great Britain R&M 3427, 1966.
8. Inoue, M. and Kaneko, K.: Effect of Trailing Edge Thickness on the Cascade Performance of Circular-Arc Blade. 2nd Int. JSME Symposium Proceedings, Tokyo, September 1972.
9. Tsien, H. S. and Finston, M.: Interaction Between Parallel Streams of Subsonic and Supersonic Velocities. Journal of the Aeronautical Sciences, Vol. 16, No. 9, Sept. 1949.
10. Lighthill, M. G.: Reflection at a Laminar Boundary Layer of a Weak, Steady Disturbance to a Supersonic Stream, Neglecting Viscosity and Heat Conduction. Quarterly Journal of Mechanics and Applied Mathematics, Vol. III, 1950.
11. Crocco, L. and Lees, L.: A Mixing Theory for the Interaction Between Dissipative Flows and Nearly Isentropic Streams. Journal of the Aeronautical Sciences, Vol. 19, No. 10, Oct. 1952.
12. Stewartson, K.: Multistructural Boundary Layers. Advances in App. Mech., Vol. 14, 1974, pp. 145-239.
13. Stewartson, K., Williams, P. G.: Self Induced Separation. Proc. Roy. Soc. Ser. A, Vol. 312, 1969, pp. 181-206.
14. Lighthill, M. J.: On Boundary Layers and Upstream Influence. Proc. Roy. Soc. Ser. A, Vol. 217, 1953, pp. 478-507.
15. Davis, R. T., Werle, M. J.: Numerical Methods for Interacting Boundary Layers. Proceedings of the 1976 Heat Transfer and Fluid Mechanics Institute, Stanford University Press, 1976, pp. 317-339.
16. Briley, W. R., McDonald, H.: Numerical Predictions of Incompressible Separation Bubbles. J. Fluid Mech., Vol. 69, Part 4, 1975, pp. 631-656.
17. Williams, J. C., III: Incompressible Boundary-Layer Separation. Ann. Rev. Fluid Mechanics, 1977.
18. Carter, J. E. Inverse Solutions for Laminar Boundary-Layer Flows with Separation and Reattachment. NASA Technical Report, NASA TR R-447, November 1975.

19. Klineberg, J. M., Steger, J. L.: On Laminar Boundary-Layer Separation. AIAA Paper 74-94, 1974.
20. Reyhner, T. A., Flugge-Lotz, I.: The Interaction of a Shock Wave With a Laminar Boundary Layer. Int. J. Non-Linear Mech., Vol. 3, No. 2, June 1968, pp. 173-199.
21. Levy, R., Shamroth, S. J., Gibeling, H. J., and McDonald, H.: A Study of the Turbulent Shock Wave Boundary Layer Interaction, AFFDL-TR-76-163, Feb. 1977.
22. Spaid, F. W., and Frishett, J. C.: Incipient Separation of a Supersonic Turbulent Boundary Layer, Including Effects of Heat Transfer. AIAA Journal, Vol. 10, No. 7, July 1972.
23. Briley, W. R., and McDonald, H.: Solution of the Multidimensional Compressible Navier-Stokes Equations by a Generalized Implicit Method. Journal of Computational Physics, Vol. 24, Aug. 1977, p. 372.
24. Douglas, J., and Gunn, J. E.: A General Formulation of Alternating Direction Methods. Numerische Math., Vol. 6, 1964, pp. 428-453.

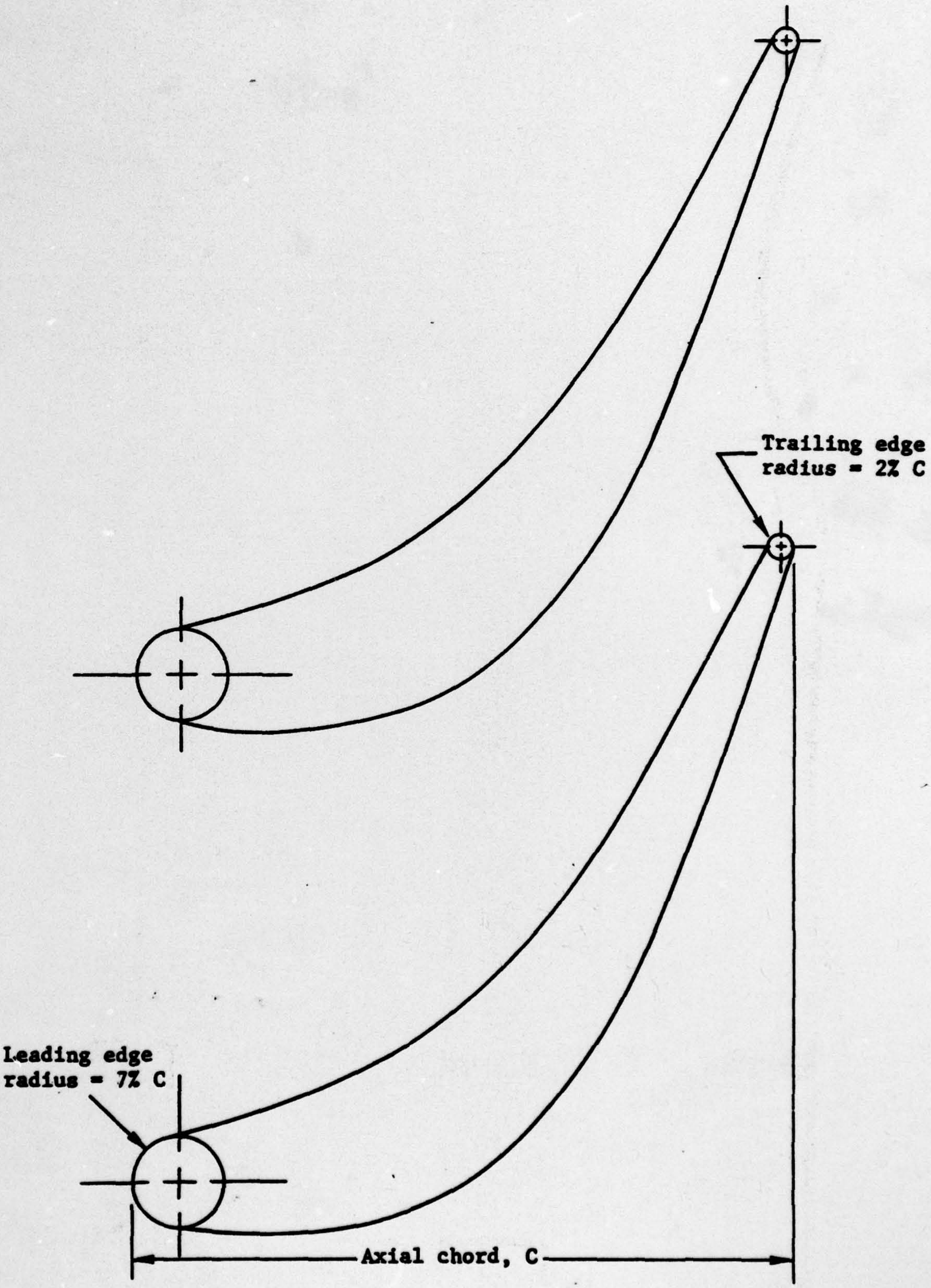

Figure 1. Typical modern turbine blade.

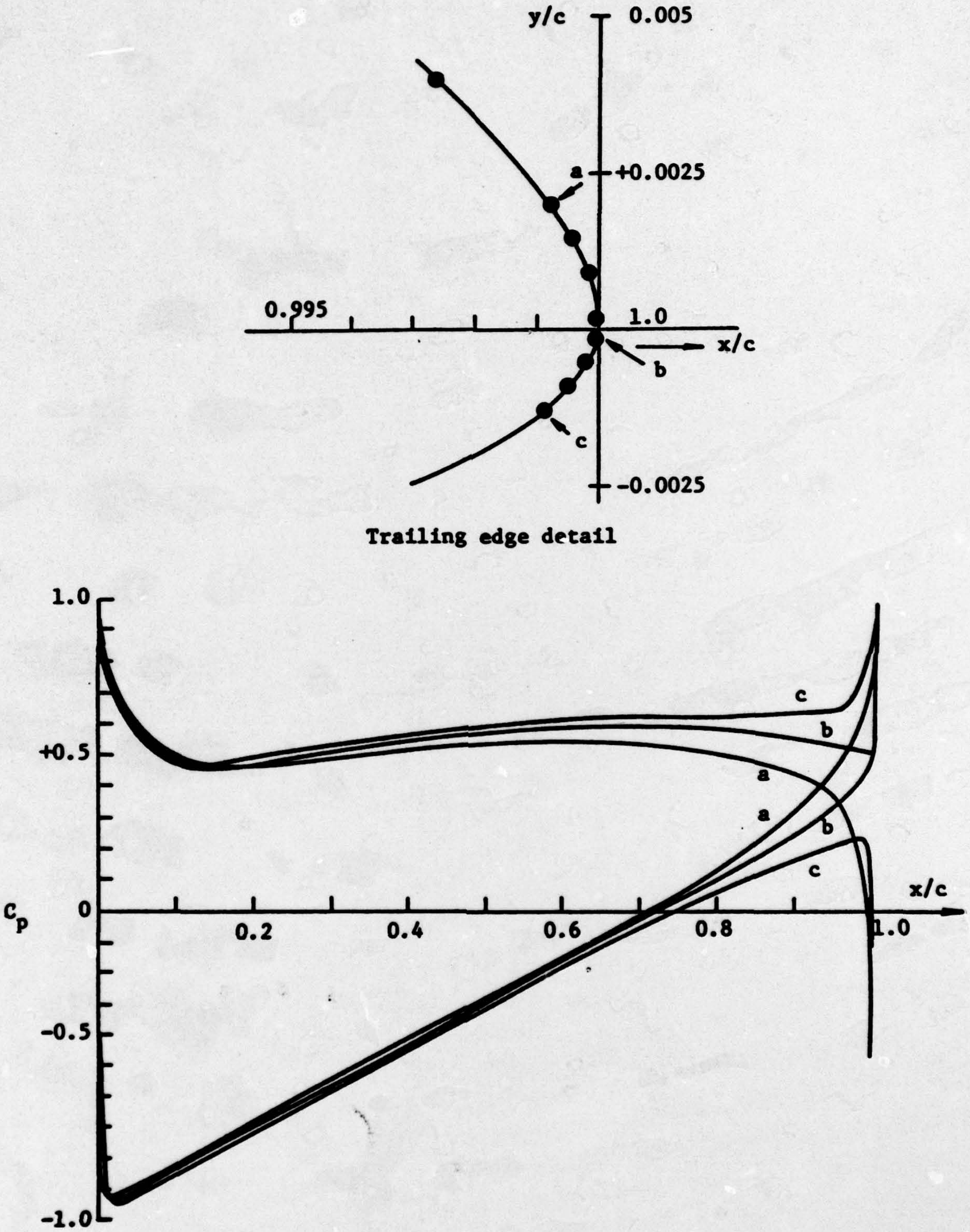

Figure 2. Effect of variation of rear stagnation point on pressure distribution predicted according to Gostelow (Ref. 3).

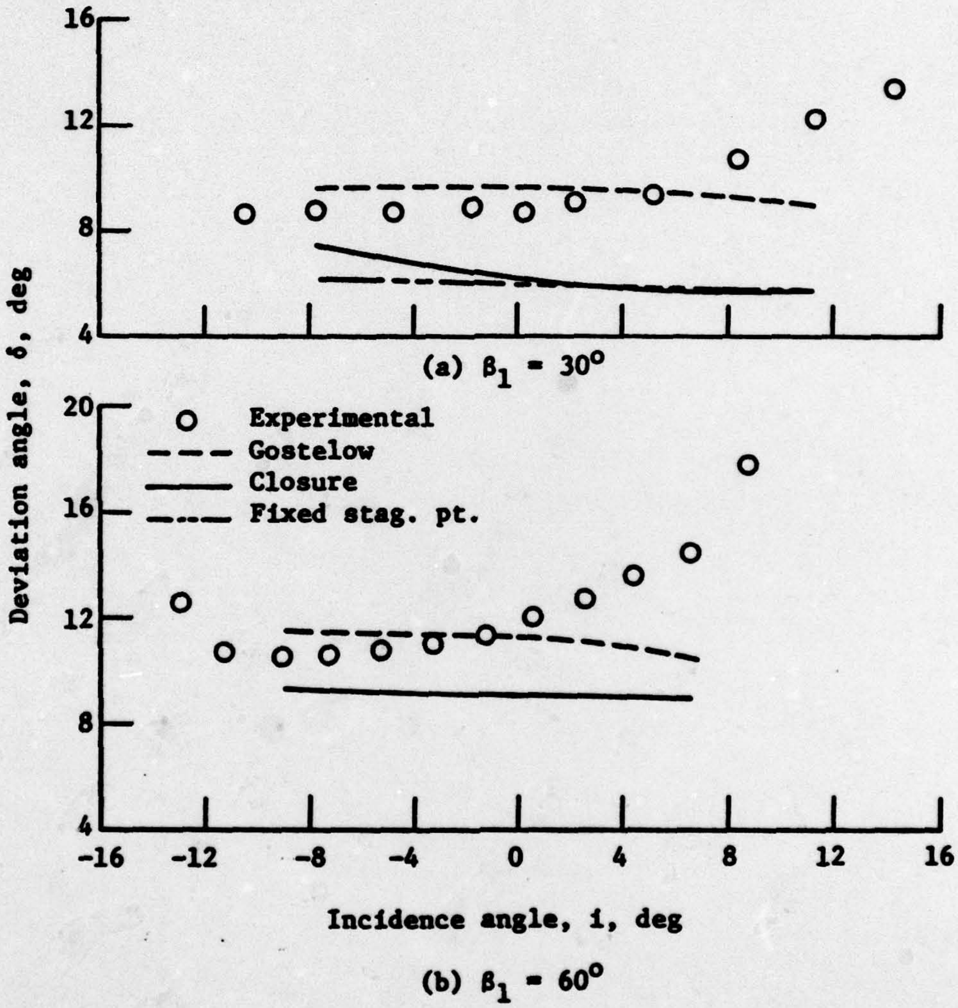

Figure 3. Deviation angle comparisons of Ref. 4 reproduced from Gostelow (Ref. 3).

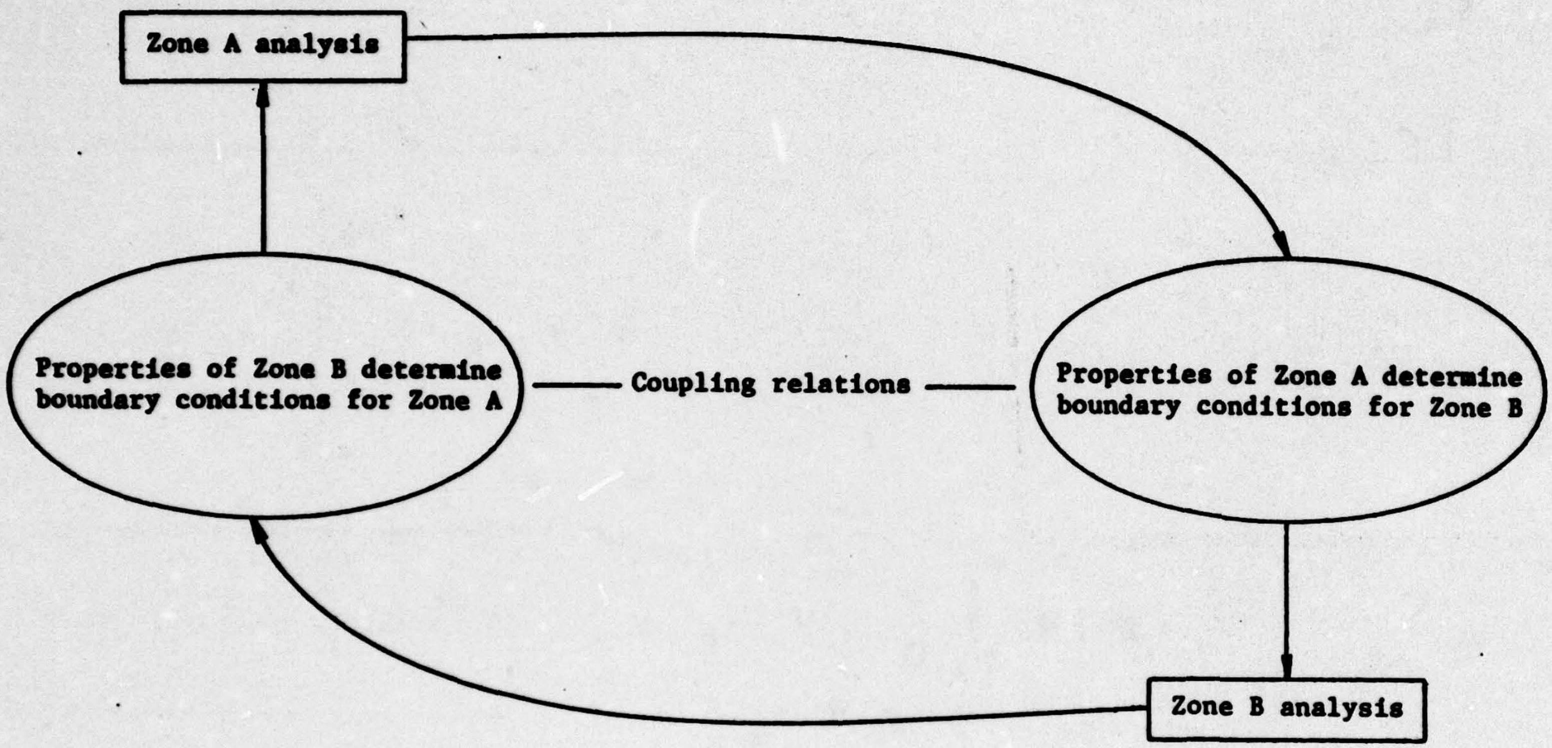

Figure 4. Flow of information provided by coupling relations.

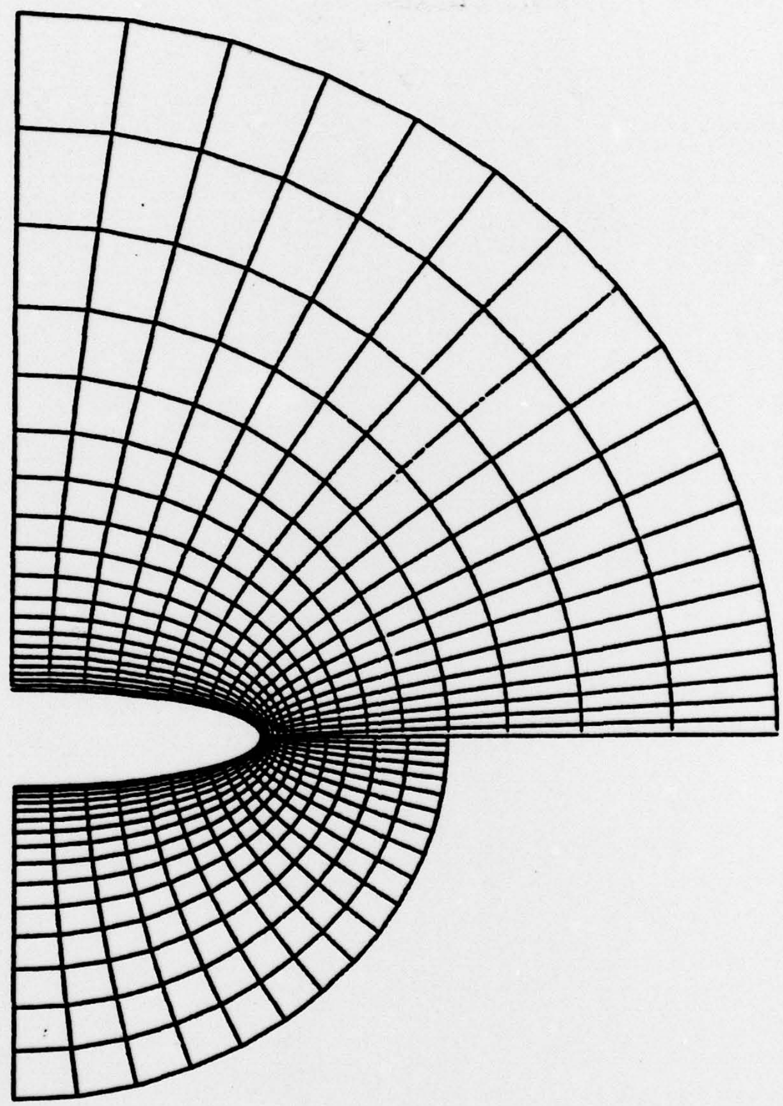

Figure 5. Comparison of computational grids for larger and smaller domains.

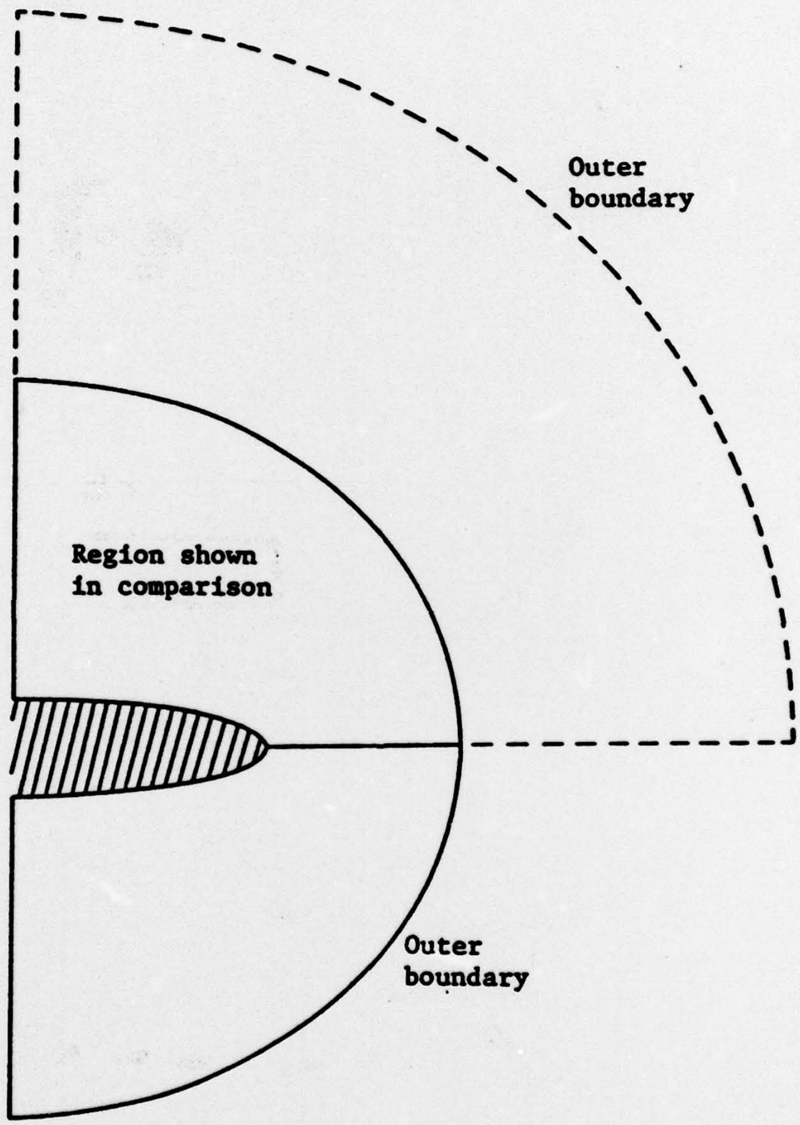

Figure 6. Larger and smaller computational domains for comparisons shown in Figures 7-12.

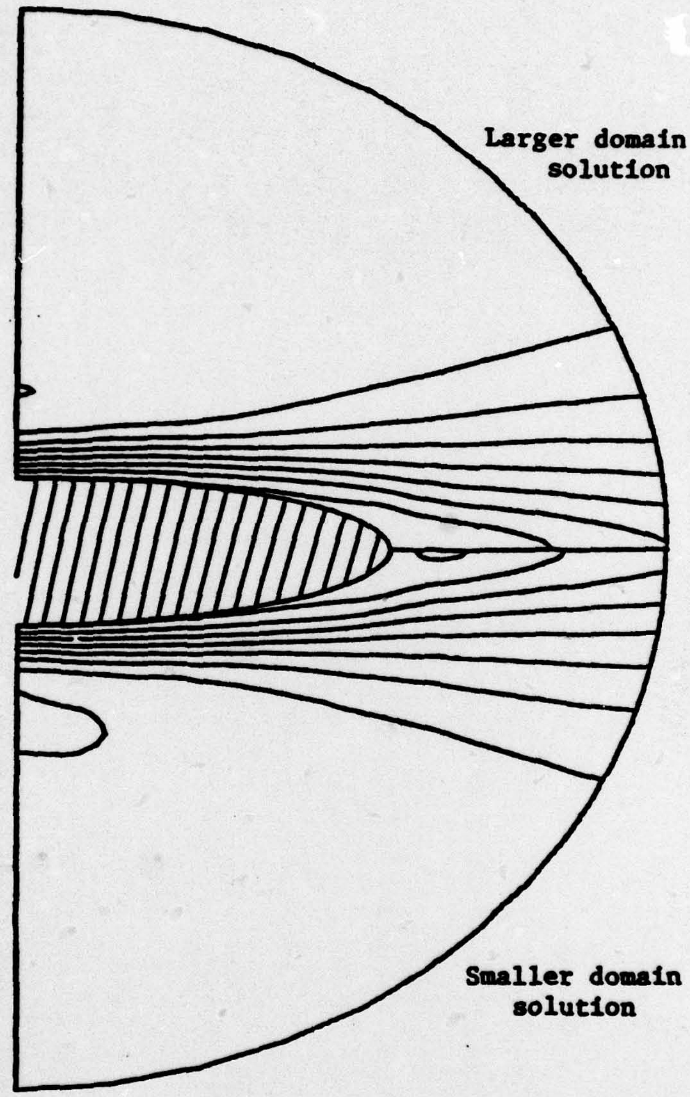

Figure 7. Comparison of horizontal velocity contours.

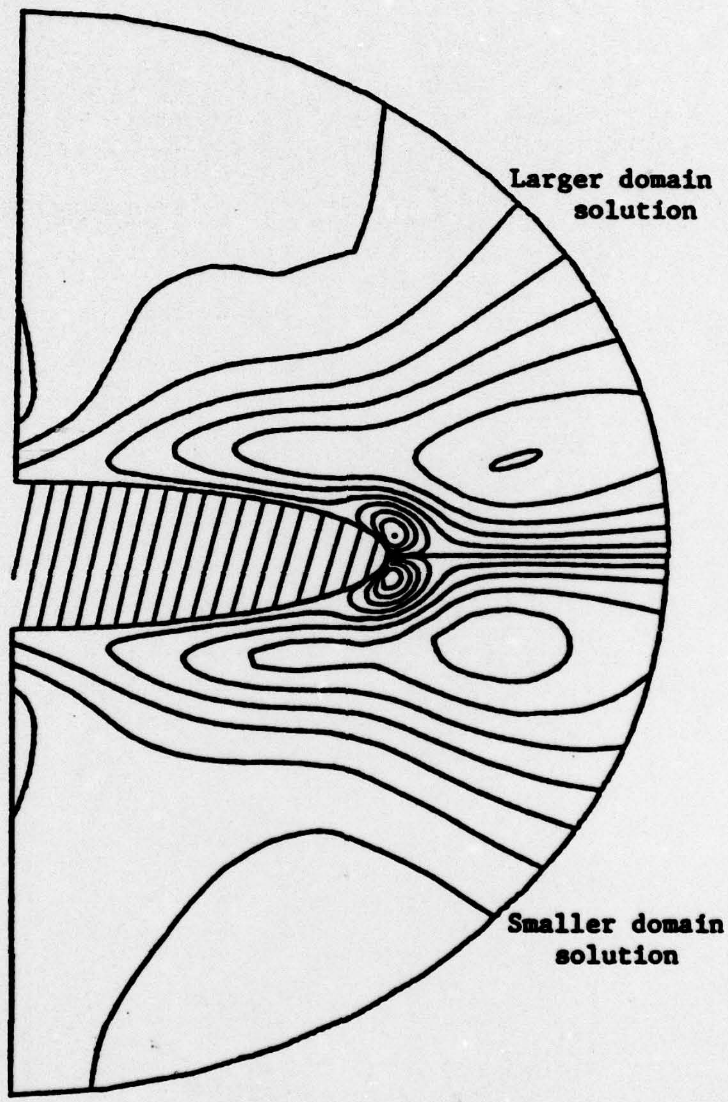

Figure 8. Comparison of vertical velocity contours.

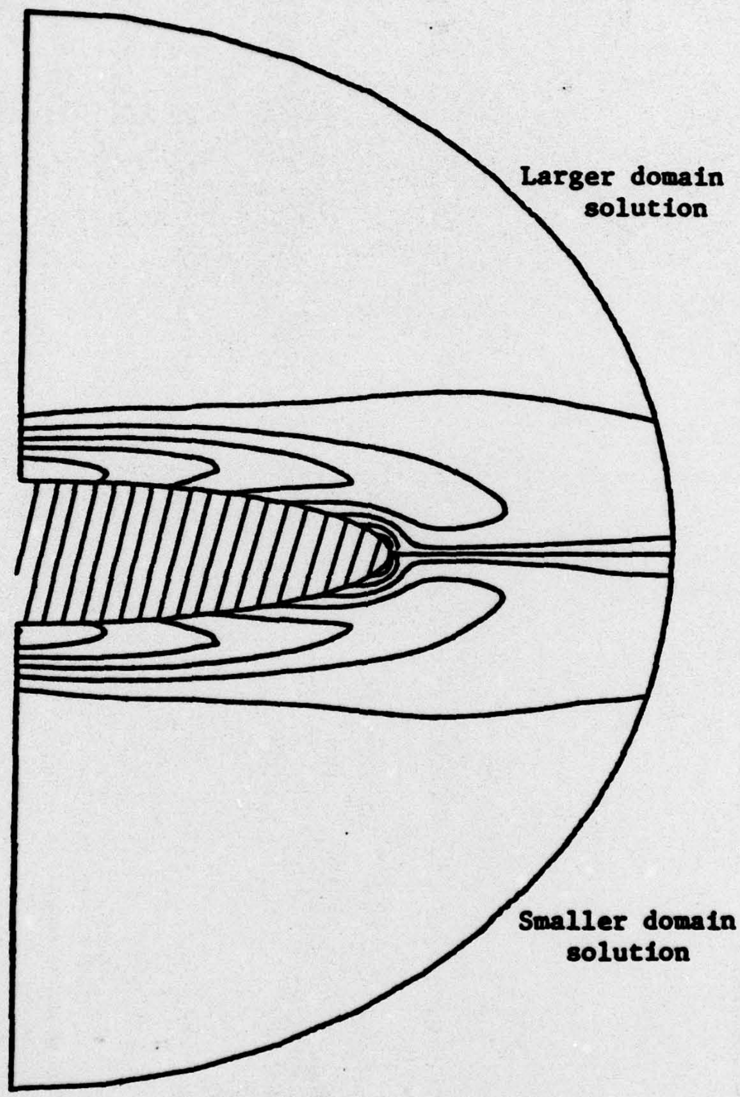

Figure 9. Comparison of vorticity contours.

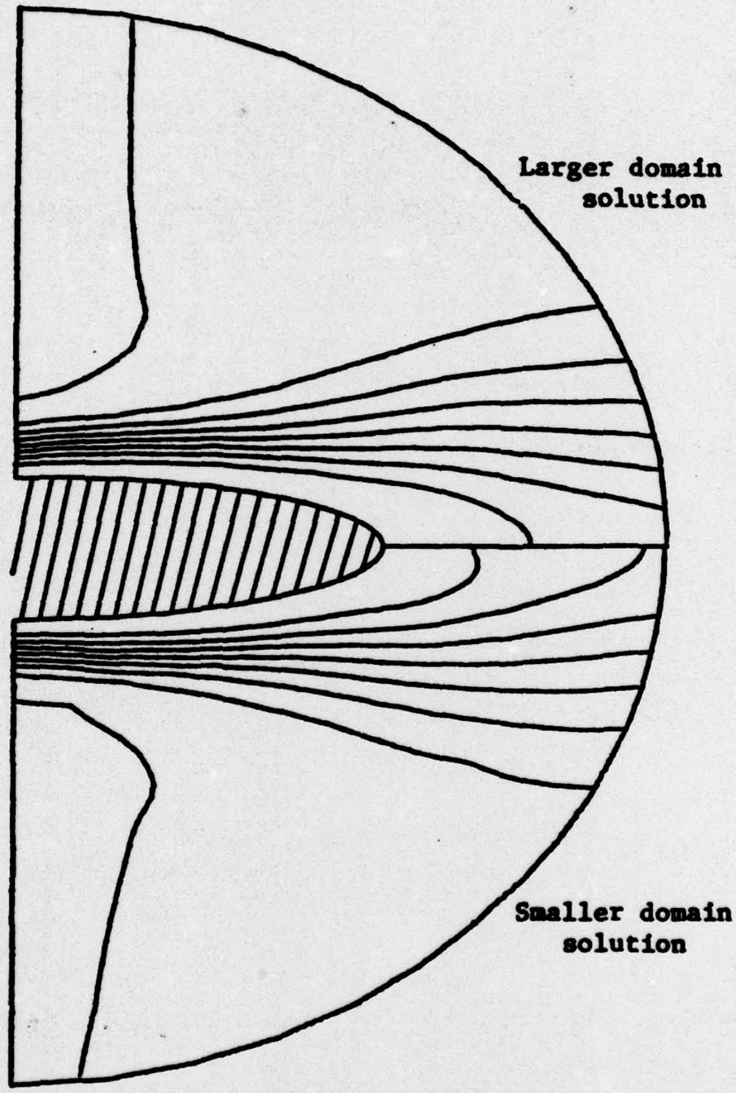

Figure 10. Comparison of stagnation pressure contours.

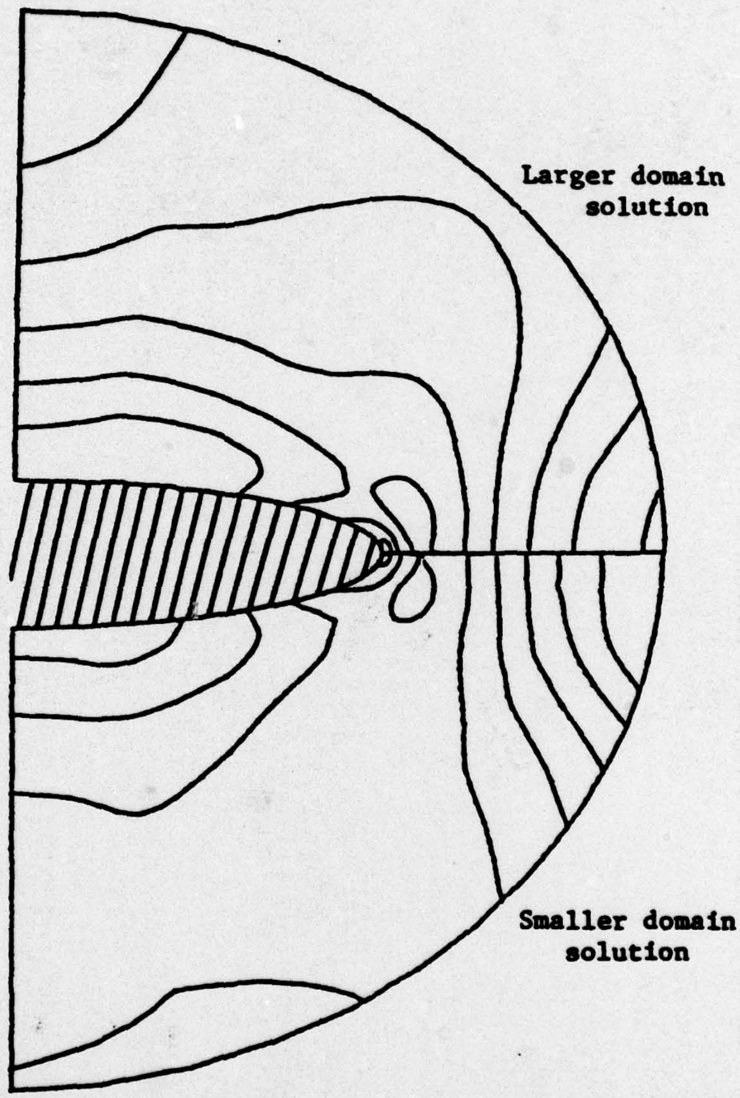

Figure 11. Comparison of static pressure contours.

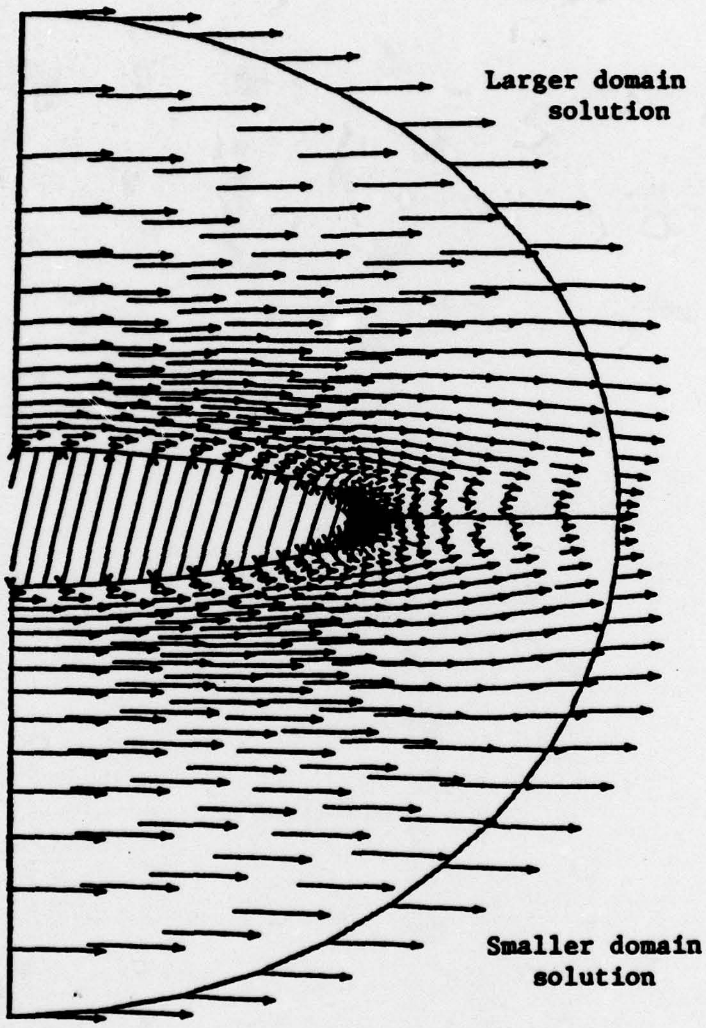

Figure 12. Comparison of velocities.

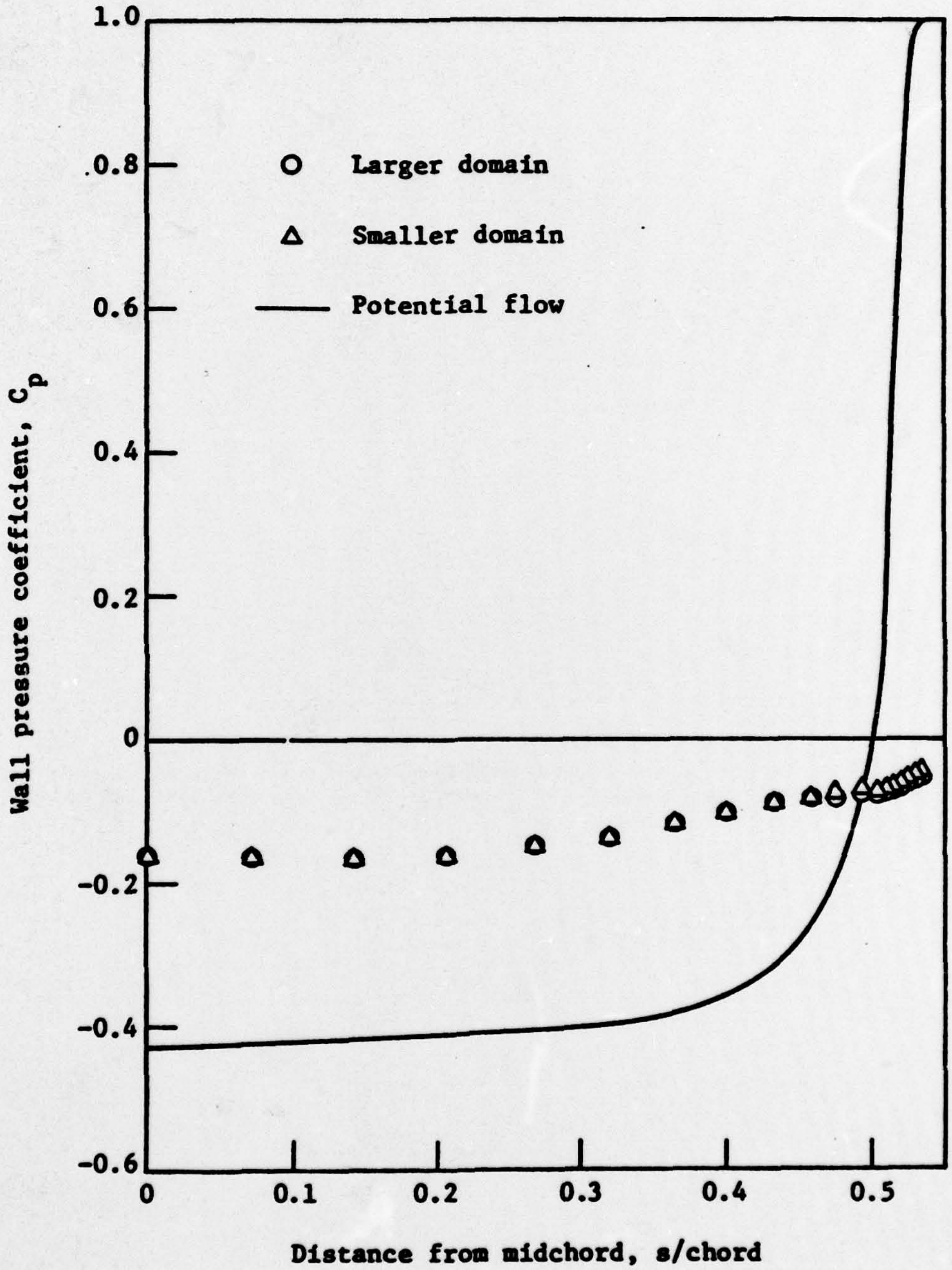

Figure 13. Comparison of computed wall pressure coefficients.

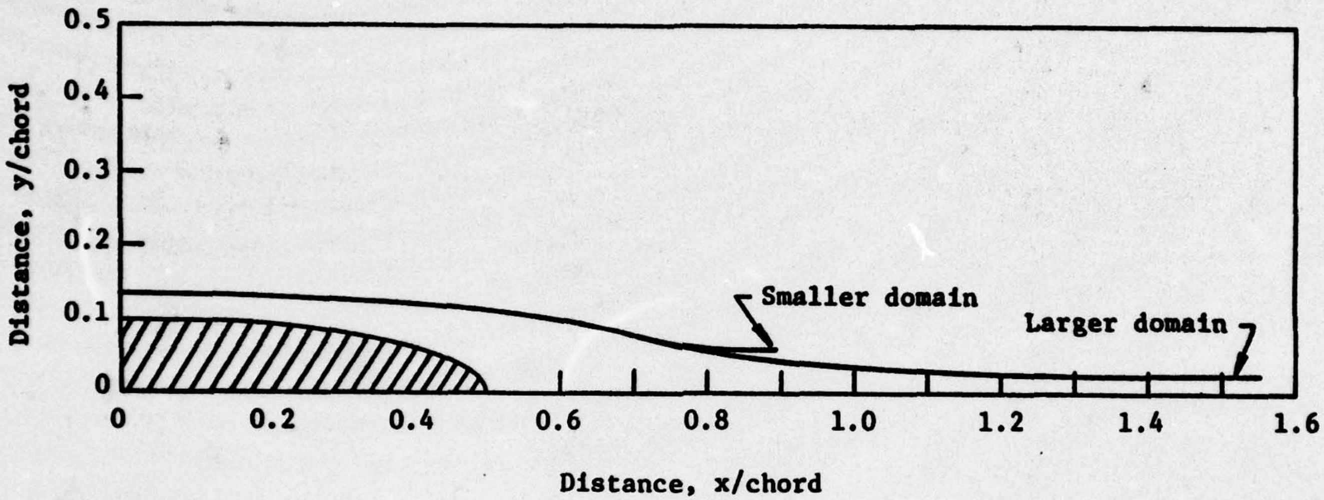

Figure 14. Comparison of computed interaction streamlines.

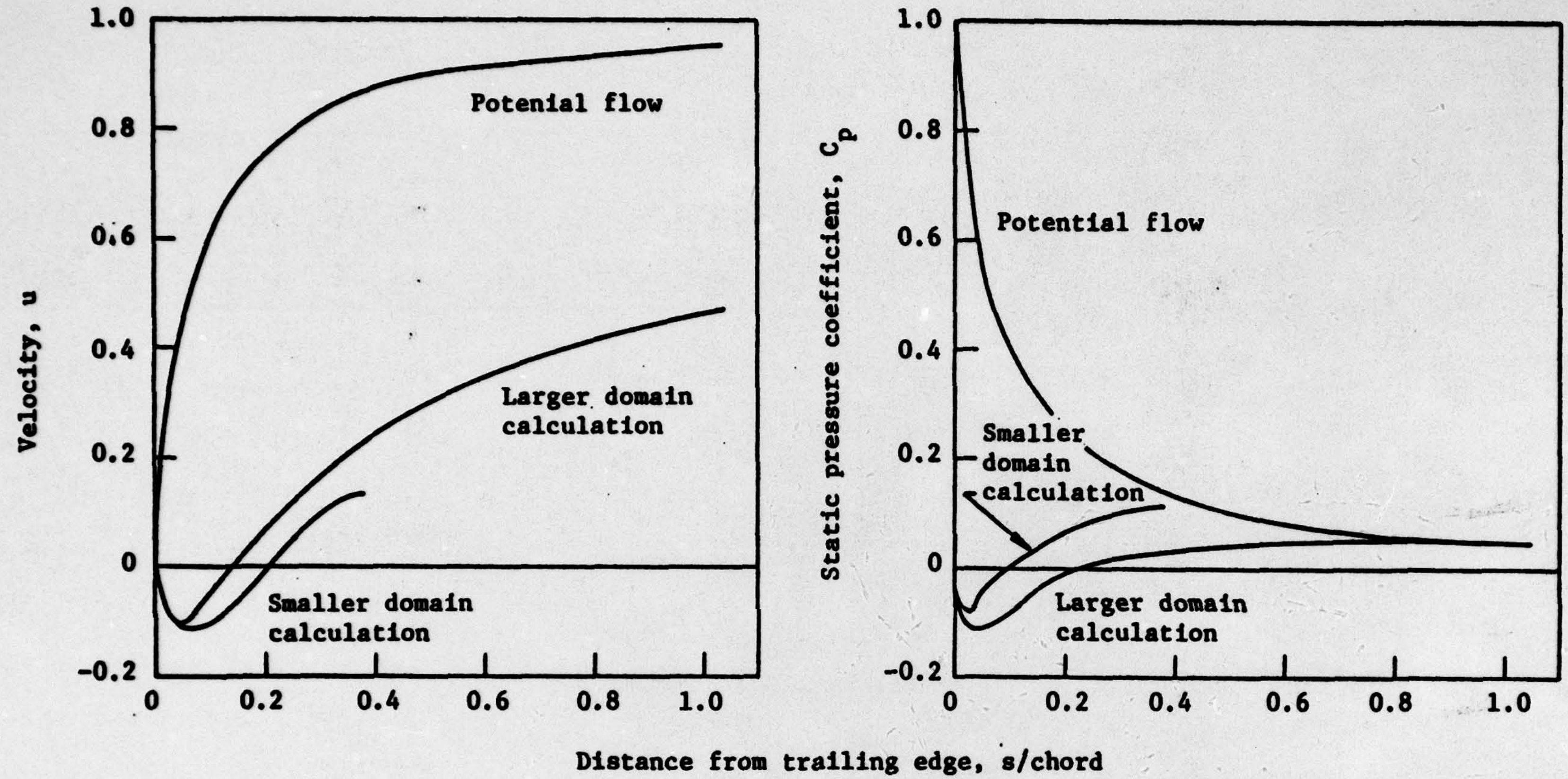

Figure 15. Comparison of computed wake centerline properties.